Second year Higher Secondary Examination

PART III

MATHEMATICS (SCIENCE)

MATRIX ALGEBRA

Maximum: 30 (Scores)

HSE II
2015 TIME: 1 Hour

പ്രിയപ്പെട്ട കുട്ടികളെ , Matrix ടെസ്റ്റ് പേപ്പർ എഴുതിക്കാണുമെന്ന് വിശ്വസിക്കുന്നു. അതിന്റെ Answer key ഇതിനോടൊപ്പം ഉൾപ്പെടുത്തുന്നു. ടെസ്റ്റ് പേപ്പർ എഴുതാത്ത കുട്ടികൾ answer കീ നോക്കാതെ തന്നെ ടെസ്റ്റ് എഴുതുമല്ലോ?

Solution for Matrix_Test_1

1. Let A = $\begin{bmatrix} 3 & 6 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ and C = $\begin{bmatrix} 3 & 6 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ Find the matrix B such that 2A+B=3C

$2A + B = 3C \Rightarrow B = 3C - 2A$

$$= 3\begin{bmatrix} 3 & 6 & 5 \\ 6 & 7 & 8 \end{bmatrix} - 2\begin{bmatrix} 3 & 6 & 5 \\ 6 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 9 & 18 & 15 \\ 18 & 21 & 24 \end{bmatrix} - \begin{bmatrix} 6 & 12 & 10 \\ 12 & 14 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$ (2)

2. $AB = \begin{bmatrix} 5 & 3 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 5\times 2 + 3\times 4 + 10\times 6 \end{bmatrix} = \begin{bmatrix} 10+12+60 \end{bmatrix} = \begin{bmatrix} 82 \end{bmatrix}$ (2)

3. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = A^2 - 2A - 3I$, find f(A)

$A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ (1)

$2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$; $3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. (1)

$f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$. (1)

4. Find the values of x,y and z from the following equations: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ (3)

$x + y + z = 9$ ....(1)
$x + z = 5$ ....(2)
$y + z = 7$ ....(3)

From (2):

$z = 5 - x$ ....(4)

In (1) we have,

$x + y + 5 - x = 9 \Rightarrow y = 9 - 5 = 4$

In (3) we have, $4 + z = 7 \Rightarrow z = 7 - 4 = 3$

In (4) we have $x = 5 - z = 5 - 3 = 2$

$\therefore\ x = 2\ ;\ y = 4\ \ and\ \ z = 3$

5. A man buys 8 dozen mangoes, 10 dozen apples and 4 dozens bananas. Mangoes cost Rs.18 per dozen, apples Rs. 9 per dozen and bananas Rs.6 per dozen. Represent the quantities bought by a row matrix and the prices by a column matrix and obtain the total cost.

   Let A = matrix of the quantities bought

   $= \begin{bmatrix} 8 & 10 & 14 \end{bmatrix}$

   Let B = matrix of the cost = $\begin{bmatrix} 18 \\ 9 \\ 6 \end{bmatrix}$

   The total cost = $\begin{bmatrix} 8 & 10 & 4 \end{bmatrix} \begin{bmatrix} 18 \\ 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 8 \times 18 + 10 \times 9 + 4 \times 6 \end{bmatrix}$

   $= \begin{bmatrix} 144 + 90 + 24 \end{bmatrix} = \begin{bmatrix} 258 \end{bmatrix}$

   $\therefore$ Total cost = Rs.258/- (3)

6. Let $A = \begin{bmatrix} 2 & 4 \\ -1 & 1 \end{bmatrix}$. Using elementary row transformations, find the inverse of A. (3)

$A = IA$

$$\begin{bmatrix} 2 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 5 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$$

$R_2 \to \frac{1}{6} R_2$

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1}{6} & \frac{2}{6} \end{bmatrix} A$$

$R_1 \to R_1 - 5R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{5}{6} & 1-\frac{10}{6} \\ \frac{1}{6} & \frac{2}{6} \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & -\frac{4}{6} \\ \frac{1}{6} & \frac{2}{6} \end{bmatrix} A$$

$$I = \frac{1}{6}\begin{bmatrix} 1 & -4 \\ 1 & 2 \end{bmatrix} A$$

$$IA^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -4 \\ 1 & 2 \end{bmatrix} AA^{-1}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -4 \\ 1 & 2 \end{bmatrix} \qquad [IA^{-1} = A^{-1}\ ;\ AA^{-1} = I]$$

7. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix}$

a) order of AB $= 2 \times 2$ (1)

b) $A' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & -1 \end{bmatrix}$ and $B' = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 6 \end{bmatrix}$ (2)

c) $AB = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -7 \\ 8 & 4 \end{bmatrix}$ (2)

$(AB)' = \begin{bmatrix} 9 & 8 \\ -7 & 4 \end{bmatrix}$ ………….. (1)

$$B'A' = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -7 & 4 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots..(2)$$

From (1) and (2) we have $(AB)' = B'A'$

8. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$. Consider the following statement:

$P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for all $n \in N$

a) $P(1): A^1 = \begin{bmatrix} 1+2\times 1 & -4\times 1 \\ 1 & 1-2\times 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ (1)

b) Assume that P(k) is true.

Then $P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

To show that P(k+1) is true, (4)

$$P(k+1): A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$LHS = A^{k+1} = A^k.A = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} (1+2k)3 + -4k.1 & (1+2k)\times -4 + -4k(-1) \\ k.3 + (1-2k).1 & k(-4) + (1-2k)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix} = RHS$$

Hence P(k+1) is true.

$\therefore$ P(n) is true for all $n \in N$.

9. Express $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices. (4)

$$A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

Let $P = \frac{1}{2}\left(A + A^T\right)$, is symmetric

Let $Q = \frac{1}{2}\left(A - A^T\right)$, is skew-symmetric

Now $P + Q = \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\begin{bmatrix} 6 & 6 & -2 \\ -4 & -4 & 2 \\ -8 & -10 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$, a square matrix.