

Second year Higher Secondary Examination

II YEAR
FIRST TERM 2016

TIME: 1 Hour
Cool-off time: 15 minutes

PART III

MATHEMATICS (SCIENCE) – Chapters 1,2 and 3

Maximum: 20 (Scores)

1. (a) Show that the relation R in R defined as $R = \{(a,b): a \leq b\}$ is reflexive, transitive but not Symmetric. (2)
- (b) Let $f : N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer. (3)
- (c) Consider $f : R \rightarrow R$ given by $f(x) = 5x + 2$.
- a) Show that f is one-to-one. (1)
- b) Is f invertible? Find f^{-1} . (2)

OR

Let $*$ be a binary operation on the set Q of rational numbers as $a * b = \frac{ab}{4}$. Check whether the operation is a) commutative, b) associative and c) identity. (3)

2. (a) If $A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$, prove that $A^2 - 4A - 7I = 0$. (2)
- (b) If $A(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, Show $A(x) \cdot A(y) = A(x+y)$ (2)
- (c) Using elementary row transformations, find the inverse of $\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ (2)
3. (a) Find the principal values of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$
- (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $-\frac{\pi}{4}$ (1)
- (b) Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$ (3)
4. (a) Show that $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \frac{\pi}{4} + \frac{x}{2}$. (3)

OR

Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x, x > 0$ (3)