HIGHER SECONDARY EXAMINATION

II YEAR — TIME: $2\frac{1}{2}$ Hours

SECOND TERM 2015 — Cool-off time: 15 minutes

PART III

MATHEMATICS (SCIENCE)

Maximum: 80 (Scores)

GENERAL INSTRUCTIONS TO CANDIDATES:

- There is a 'Cool-off time' of 15 minutes in addition to the writing time of $2\frac{1}{2}$ hours.
- You are not allowed to write your answers or to discuss anything with others during the 'Cool-off time'.
- Use 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

1. The total profit y (in rupees) of a drug company from the manufacture and sale of x bottles of drug is given by $y = \frac{-x^2}{300} + 2x - 50.$

   i) How many bottles of drug must the company sell to obtain the maximum profit? (4)

   ii) What is the maximum profit? (2)

   OR

   i) The length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum. (6)

2. i) Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R. (3)

   ii) Find differential to approximate $(26)^{\frac{1}{3}}$. (4)

   OR

   i) Find the intervals in which the function *f* given by $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing. (4)

   ii) Find approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$. (3)

3. Evaluate:

   i) $\int \frac{dx}{\left(e^x + e^{-x}\right)^2}$ (3)

(P.T.O)

ii) $\int \frac{x-1}{(x-2)(x-3)} dx$ (3)

iii) $\int x^2 e^x dx$ (3)

iv) $\int \frac{(1+\log x)^2}{x} dx$ (3)

4. Evaluate:

i) $\int \frac{1}{e^x - 1} dx$ (3)

ii) $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx.$ (2)

iii) $\int \frac{dx}{\sqrt{9-25x^2}}$ (3)

iv) $\int \frac{dx}{x^4 - 1}$ (4)

5. i) The value of $\int e^x [f(x) + f'(x)] dx =$ ………………… (2)

ii) Using the above result, find $\int \frac{x}{(1+x)^2} .e^x dx$ (3)

6. i) Prove that the points whose position vectors are given by $2\vec{i} - \vec{j} + \vec{k},\ \vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} - 4\vec{k}$

form a right-angled triangle. (3)

ii) Find the value of λ such that the vectors $\vec{a} = 2\vec{i} + \lambda\vec{j} + \vec{k}$ *and* $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ are perpendicular. (2)

7. (a) If $\bar{a} = 3\hat{i} - \hat{j} - 5\hat{k}$ $\bar{b} = \hat{i} - 5\hat{j} + 3\hat{k}$, show that $\bar{a} + \bar{b}$ *and* $\bar{a} - \bar{b}$ *are* perpendicular. (3)

(b) Given the position vector of three points $\vec{A} = (\hat{i} - \hat{j} + 2\hat{k})$; $\vec{B} = (4\hat{i} + 5\hat{j} + 8\hat{k})$; $\vec{C} = (3\hat{i} + 3\hat{j} + 6\hat{k})$

(i) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (3)

(ii) Prove that A,B,C are collinear points. (2)

8. i) Find the equation of the plane passing through the intersection of the two planes

$\vec{r}.(i + j + k) = 6$ *and* $\vec{r}.(2i + 3j + 4k) = -5$ and through the point (1,1,1). (3)

ii) Find the distance from point (2, 1, –3) to the plane $2x - 3y + 6z = 2$ (3)

9. i) Reduce the equation of a plane $3x - 2y + 4z = 5$ into intercept form. And obtain its intercepts. (3)

i) Find the angle between the line $\vec{r} = 2i + 3j + 4k + \lambda(\vec{i} + 3\vec{j} + 4\vec{k})$ and the plane $\vec{r}.(4i + j - 3k) = 1$ (3)

10. Evaluate $\int_0^1 e^x$ dx as the limit of a sum. (4)

11. i) Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ (3)

ii) Let the two skew lines are $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ *and* $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.

a) Find the dr's of the above lines (2)

b) Using Cartesian formula, find the shortest distance between the lines. (4)
