Second year Higher Secondary Examination

II YEAR — TIME: $2\frac{1}{2}$ Hours

FIRST TERM 2015 — Cool-off time: 15 minutes

PART III

MATHEMATICS (SCIENCE)

Maximum: 80 (Scores)

GENERAL INSTRUCTIONS TO CANDIDATES:

- There is a 'Cool-off time' of 15 minutes in addition to the writing time of $2\frac{1}{2}$ hours.
- You are not allowed to write your answers nor to discuss anything with others during the 'Cool-off time'.
- Use 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

1. (a) Let $f : A \to B$ be a function, then f^{-1} exists if

   (i) f is one-one. (ii) f is onto. (iii) f one-one but not onto.

   (iv) f is one-one and onto. (1)

   (b) Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$ (3)

2. (a) Consider a binary operation ^ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b = \min(a, b)$. Write the operation table of the operation ^. (3)

   (b) Show that the signum function $f : R \to R$ is given by $f(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ -1, if\ x > 0 \end{cases}$ is neither one-one nor onto. (3)

   (c) Consider $f : R \to R$ given by $f : R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$.

   The inverse of f (4)

3. If $X=\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}$ then X^n for $n \in N$ is equal to

(a) $2^{n-1}X$ (b) $n^2 X$ (c) nX (d) $2^{n+1}X$ (e) $2^n X$ (1)

4. (a) If A = $\begin{bmatrix}3 & 2\\ 5 & 1\end{bmatrix}$, prove that $A^2 - 4A - 71 = 0$. (3)

(b) Let $A=\begin{bmatrix}2 & 2 & 5\\ 4 & 1 & 3\\ 0 & 6 & 7\end{bmatrix}$. Express A as the sum of a symmetric and a skew symmetric matrix. (3)

5. (a) Using the properties of determinants, prove that

$$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$$ (4)

OR

Evaluate the value of the determinant: $\begin{vmatrix}1^2 & 2^2 & 3^2\\ 2^2 & 3^2 & 4^2\\ 3^2 & 4^2 & 5^2\end{vmatrix}$ (4)

(b) If A = $\begin{bmatrix}1 & 1 & 5\\ 0 & 1 & 3\\ 0 & -1 & -2\end{bmatrix}$ what is the value of $|3A|$? (3)

(c) Find the equation of the line joining the points $(1,2)$ and $(-3,-2)$ using determinants. (3)

6. Consider the following system of equations:

$x+y+z=6$

$x-y+z=2$

$2x+y+z=1$

i) Express this system of equations in the standard form $AX=B$. (1)

ii) Prove that A is non-singular. (2)

iii) Find the values of x,y and z satisfying the above system of equations. (3)

6. (a) Find the principal values of $\sin^{-1}\left(-\frac{1}{2}\right)$

(i) $\frac{\pi}{6}$ (ii) $-\frac{\pi}{6}$ (iii) $\frac{\pi}{3}$ (iv) $-\frac{\pi}{3}$ (1)

(b) Prove that $\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{63}{16}\right)$ (3)

7. (a) Show that $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\frac{\pi}{4}+\frac{x}{2}$. (3)

(b) Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x, x>0$ (3)

8. Consider the function $f(x)=\begin{cases}3ax+b & if\ x>1\\ '' & if\ x=1\\ 5ax-2b & if\ x<1\end{cases}$

(a) Find $\lim_{x\to1^-} f(x)$ *and* $\lim_{x\to1^+} f(x)$ (2)

(b) Find the constants a and b if $f(x)$ is continuous at $x=1$ (4)

(c) At the point x = 0, the function f (x) = |x| is

(a) continuous, but not differentiable (b) differentiable, but not continuous

(c) continuous and differentiable (d) neither continuous not differentiable (1)

9. (a) Find $\frac{dy}{dx}$, if

(i) If $\sin(xy)+y\cos^2 x=1$ (3)

(ii) Find $\frac{d^2y}{dx^2}$ if $x=a(\theta-\sin\theta)$; $y=a(1-\cos\theta)$. (3)

(iii) If $y=e^{m\cos^{-1}x}$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$ (3)

10. (a) If $y=a^x$, *then* $\frac{dy}{dx}$ is

(a) a^x b) $a^x log\ x$ c) $x\,a^{x-1}$ d) $\frac{dy}{dx}=1$ (1)

(b) If $x^3+y^3=3axy$, *find* $\frac{dy}{dx}$ (3)

(c) Use differential to approximate the value of $(26)^{\frac{1}{3}}$. (3)

11. (a) Which of the following is an always increasing function?

(a) logx b) tanx c) cotx d) x (1)

(b) Verify Rolle's theorem for the function $f(x)=x^2+2x-8, x\in[-4,2]$ (3)

12. (a) A stone is dropped into a quiet lake and waves move in circles at the speed of 5cm/sec. At the instant when the radius of the circular wave is 8cm, how fast is the enclosed area increasing? (3)

(b) Find the equation of the tangent line to the curve $y=x^2-2x+7$, which is perpendicular to the line $5y-15x=13$. (3)
